# Magnet Sorting for the TTF-FEL Undulator using Simulated Annealing

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**Abstract**

In order to get the field quality required for the undulator of the TESLA Test Facility Free Electron Laser (TTF-FEL), two steps have been foreseen. The first step is to sort the magnets in such a way that the resulting field quality is as good as possible with the magnets at hand. The second step is to use techniques such as pole height adjustment and/or shimming to tune both $x$ and $y$ field components to the required specifications. The main purpose of the magnet sorting is to minimize corrections in the second step. Therefore, two transverse components of the field, ideally zero, are minimized by this procedure.

In this paper we outline the magnet sorting procedure using simulated annealing. For the three undulator segments for phase I of the TTF-FEL geometrical constrains were properly taken into account. The results obtained for the magnets being used are presented.

## 1. Introduction

The goal of the TTF-FEL [1] is to supply future users with VUV radiation, down to a wavelength of 6 nm. It will operate as a single pass device starting from noise, the so-called Self Amplified Spontaneous Emission (SASE) regime [2,3]. The electron beam will be supplied by a low emittance photo cathode gun and accelerated by a high-gradient superconducting accelerator. In order to get the desired peak current of 2.5 kA, the electron bunches will be compressed in three stages. The radiation will be produced in an undulator with a superimposed alternating gradient (FODO) focusing structure. In order to test these components at an early stage and prove the SASE principle in this wavelength range, first tests will be performed with only three accelerating modules in place ($E_b < 390\,\mathrm{MeV}$), two bunch compressors, resulting in a peak current of 500 A, and half of the undulator consisting of three segments. Parameters are given in Table 1.

In order for the radiation to reach saturation within the given undulator length, we have to guarantee a close overlap between electron beam and radiation field. Earlier

Table 1
Undulator and electron beam parameters for the TTF FEL (Phase I).

| | |
|---|---|
| **Electron beam** | |
| Peak current | 500 A |
| Normalized rms emittance | $2\pi$ mm mrad |
| rms energy spread | 500 keV |
| average beam size | 70 $\mu$m |
| **undulator** | |
| number of segments | 3 |
| length of segment | 4.4922 m |
| period length | 27.3 mm |
| undulator peak field | 0.497 T |
| length of quadrupoles | 136.5 mm |
| number of quads./segment | 10 |
| distance between quads. | 341 mm |
| quad. gradient | 12.5 T/m |

simulations have shown that this can be achieved if the electron beam does not deviate from its ideal trajectory by more than 20% of its rms beam size, i.e. 10 to 12 $\mu$m in case of the TTF-FEL [4]. Although corrector stations are foreseen along the entire undulator to correct possible errors, one should still make the undulator quality as good as possible in order to keep corrections small.

Undulators are manufactured from individual parts like permanent magnets, soft iron poles, holders etc. which cannot be produced to be absolutely perfect. Mechanical parts contain dimensional errors typically in the order of 20 to 50 $\mu$m. By way the largest error source is the magnetic material itself. Its magnetization typically differs by $\pm 1\%$ and the angle of magnetization may vary by as much as $\pm 2 - 2.5^{\circ}$. Furthermore the material can be inhomogeneous. In order to obtain a perfect undulator field two steps seem reasonable. In the first step, the influence due to imperfect magnets is minimized by sorting magnets in such a way that errors from different magnets mutually cancel. To do so each magnet has to be measured and characterized. The magnetization, angular deviation from the nominal direction and its homogeneity is measured and characterized by a set of numbers. These numbers are input in a sorting algorithm using the method of simulated annealing [5–7]. The algorithm exchanges and flips magnets to optimize a goal function which has previously been defined and will be described in detail below. This paper deals with the sorting of the magnets for the undulator for the phase I at the TESLA Test Facility consisting of three undulator segments. Each of them is 4.4922 m long and consists of 652 magnets that have to be sorted. For the second step of optimization, pole height adjustment and shimming will be used to compensate remaining errors. These issues are not subject to this paper and have been presented elsewhere [8].

## 2. Procedure for magnet sorting

The main problem in sorting the magnets is their large number. For the TTF-FEL undulator structure, each undulator consists of 652 magnets. Checking all possible configurations would clearly be impossible. Therefore, we use a method referred to as simulated annealing. This method uses random changes in the order in which magnets are placed. Every change made is checked against a predefined function which is to be minimized.

The whole TTF-FEL phase I undulator consists of 3 segments with 652 identical magnets each that have to be sorted. Consequently, all magnets can be placed at an arbitrary position in any of the three undulator segments. For reasons of logistics (keeping track of all magnets while actually building the undulator) it has been decided to divide

the total number of magnets available (2022 in this case) in three equal parts, optimizing each undulator using only one third of the magnets. Although this limits the possible configurations, the desired accuracy can still be achieved, as will be shown in this report.

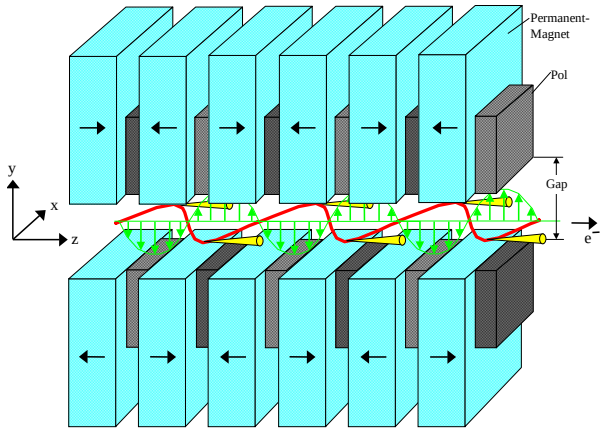



Fig. 1. *Layout of the TTF-FEL undulator.*

At a specific position in the undulator, the field direction of the magnet is given (see Fig. 1). At this position, the magnet can only be rotated by 180 degrees around the axis in the direction of the main field. This rotation will be referred to as a "FLIP" of the magnet. Flipping a magnet is the only operation that can be performed on a single magnet without changing its location. One can also interchange the position of two magnets. Since at each position the main field direction is given, this operation is unique except for a possible flip of one or both magnets. However, since it is always possible to flip afterwards, this has no consequence. For convenience, it is assumed that a flip is connected to a position rather than to a magnet. Note, that since in general there are more magnets available than needed, one can also change a magnet inside the undulator with one that was not used so far. Consequently, the final magnet distribution will not include magnets which have the worst quality. To summarize, there are two possible (fundamental) operations possible, namely flipping one magnet or changing the position of two magnets. From a given distribution of magnets, the undulator quality can sometimes only improve after two or three of the above mentioned operations. The advantage of simulated anneal is that it allows the quality to get worse within given boundaries. A number of subsequent changes is abandoned only if the quality does not improve after several operations. The procedure starts again from the best undulator quality thus far. Although the final solution does not always give the absolute best undulator quality, results will show that the improvement is adequate for the TTF-FEL undulator.

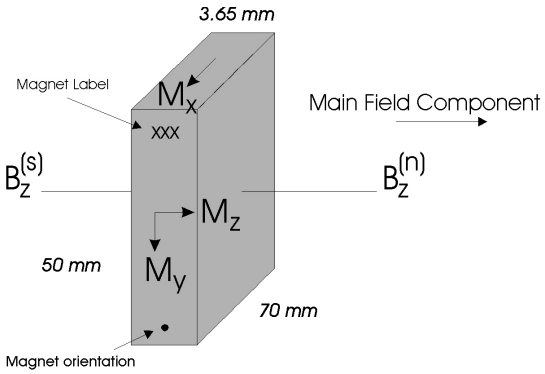



Fig. 2. *Labeling and block coordinate system for the magnets. Indicated are the three magnetization directions and the two magnetic field measurements. The magnet block size is 70 by 50 by 3.65 mm. Each magnet is specified by a magnet label, a unique number for each magnet, and an orientation to specify between the normal and flipped orientation (see text for details).*

Each block magnet has three magnetization components, $M_x$, $M_y$ and $M_z$ (see Fig. 2 for details). These components are measured for each magnet individually using a Helmholtz coil setup. For the $z$-direction, two additional measurements are made, namely

$B_z^{(n)}$ and $B_z^{(s)}$, at two geometrically equivalent positions near the north and south pole using a Hall probe mounted in a suitable fixture, in order to take into account an inhomogeneous part of the magnetization. For an ideal magnet, $M_x$ and $M_y$ are equal to zero and $B_z^{(n)} = B_z^{(s)}$. The average value of the Hall probe measurements is assumed to correspond to the measured magnetization of the magnet, i.e.

$$B_z^{(n)} \propto M_z^{(n)} = \frac{M_z}{(1 + B_z^{(s)}/B_z^{(n)})} \quad \text{and} \tag{1a}$$

$$B_z^{(s)} \propto M_z^{(s)} = \frac{M_z}{(1 + B_z^{(n)}/B_z^{(s)})} \,. \tag{1b}$$

It is easily seen that $B_z^{(n)} + B_z^{(s)} \propto M_z$. Similarly, it is assumed that $B_{(x,y)} \propto M_{(x,y)}$.

From these four values, fields can now be calculated per half period. Each magnet pair at position $i$, one of them in the lower ($\ell$) structure and in the upper ($u$), results in the following fields

$$B_{x,i} \propto (F_{\ell,i} M_{x,\ell,i} + F_{u,i} M_{x,u,i})(-1)^i \,.$$

$$B_{y,i} \propto F_{\ell,i} M_{y,\ell,i} - F_{u,i} M_{x,u,i} \,.$$

$$B_{\mathrm{und},i}^{(1)} \propto (M_{z,\ell,i}^{(n)} + M_{z,u,i}^{(s)})(-1)^{i-1} \,.$$

$$B_{\mathrm{und},i}^{(2)} \propto (M_{z,\ell,i}^{(s)} + M_{z,u,i}^{(n)})(-1)^{i} \,.$$

The factor $F$ stands for a possible flip of the magnet at this position. The fluxlines of the main magnetization $M_z$ are going through the poles between the magnets, resulting in the undulator field in the $y$-direction. The superscripts (1) or (2) stand for the two halfs of the magnet pair. The pairs (1) (or (2)) at position $i$ have to be added to (2) (or (1)) at position $i + 1$ to get the field at the pole position between the two magnets. What combination has to be added depends on the position within the undulator.

Now that all fields are given at the magnet positions ($B_x$ and $B_y$) and at the poles ($B_{\mathrm{und}}$), one is able to calculate the function that has to be optimized. One of the problems is that the proportionality constant between magnetization $M$ and field $B$ is unknown and not necessarily the same for the different field components. The magnetic field consists of an $x$-component and a $y$-component, of which the latter is an addition of the direct field ($y$-component of the magnetization of two magnets) and the undulator field (either $B_{\mathrm{und},i}^{(1)} + B_{\mathrm{und},i+1}^{(2)}$ or $B_{\mathrm{und},i}^{(2)} B_{\mathrm{und},i+1}^{(1)}$). How to add the two sources to the $y$-component is unknown. Therefore, instead of minimizing the $x$ and $y$-component of a function, three components are minimized independently, e.g. two components in $x$ and $y$-direction, and the third component which is related to the desired undulator field (denoted as $w$ for 'wiggler' in the remainder of this report).

The question to be answered is what function has to be minimized, i.e. how to specify the undulator quality. The most common way to determine the quality is to specify the rms value of the field errors, i.e. add the squares of the deviation of the peak field every half period compared to the average field. Simulations have shown, however, that this is a very poor parameter with which to determine the quality [10,11]. The main parameters that determine the performance of an undulator are the phase shake and the second field integral [4,11]. The function that has been optimized in the report is therefore the second field integral. The phase shake is expected not to be a significant problem for these small magnetic

field errors.

### 3. General layout of the program

The program used to determine the optimum configuration consists of several parts. Obviously it has to determine the input and output file names as well as some parameters to determine what undulator segment has to be sorted etc. Both input and output file have the same format, consistent with the information given by J. Pflueger [9]. The reason for this is that one possibly needs an additional optimization, either because the undulator quality is still not adequate, but also because some magnets can be damaged during the implementation into the structure, in which case some magnets have to be reordered. For this reason, one can specify if certain magnets have to be excluded from the sorting procedure. Although one could in this case in principle completely sort the magnets again according to the same procedure, it has been found that usually it is sufficient to change the magnet which is no longer available by another magnet that has not yet been used. Checking only the remaining magnets (of the order of 20) in two possible orientation is very fast in the tests performed and so far did not increase the test function (e.g. rms field error or second field integral) significantly.

The files contain a magnet number with its three magnetization components and two values of a magnetic induction, one value at the south pole and one at the north pole. In addition the position of the magnets within the total structure is given.

(1) Undulator segment number (1-3)
(2) Upper/Lower (U/L)part
(3) Module number within a segment (1-5)
(4) Magnet position within each module (1-65) [1]
(5) Magnet identification number
(6) Flipped/Normal (F/N) magnet

The first four numbers uniquely determine the magnet position within the structure, the fourth number determines which magnet is at the specified position and the last number determines the orientation of the magnet.

The core of the program consists of a few parts. One procedure determines by invoking a random number generator which of the two operations, flipping a magnet or interchanging two magnets, has to be performed. It also determines, again using random numbers, at which position a magnet has to be flipped or at what positions magnets have to be interchanged. A second procedure performes the actual operations, i.e., flipping the magnet at the position or changing magnets at two different positions. In addition, the magnetic fields are changed to take into account the changed magnets. The next procedure checks the second field integral in three directions, $x$, $y$ and $w$ (where $w$ is the main magnetic field, see the previous section). The main loop of the program now determines if the quality has been improved and if not, if the new configuration has to be excepted anyway. This procedure continues until no significant improvement is expected, e.g. no improvement has occurred after many changes of magnet position and orientation.

[1] The last magnet pair (half an undulator period) in each undulator is given separately

## 4. Results of simulations for the TTF-FEL undulator

For undulator I, a total of 674 magnets are available. The distribution of $x, y$ and $z$-components of the measured flux for these magnets is given in Fig. 3. One can also

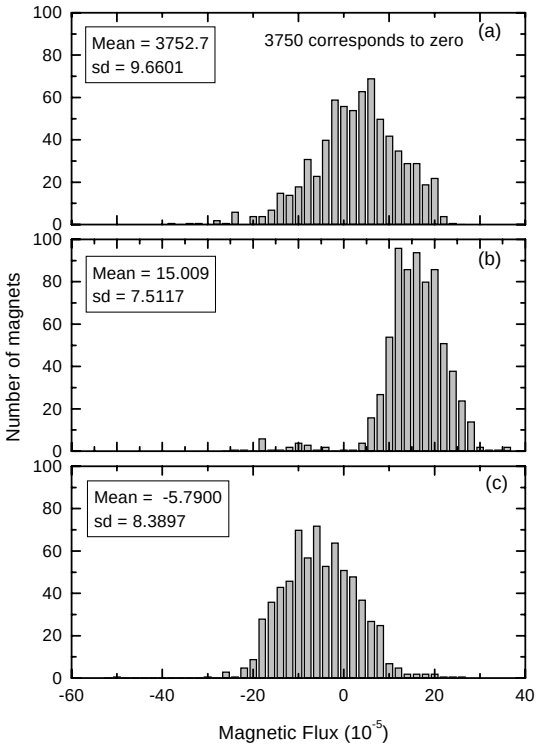



Fig. 3. *Distribution of the magnetic flux for the 700 magnets available for the first undulator. (a) gives the flux for the main field component, (b) for the magnetization in the $x$-direction, (c) for the $y$-direction. Note that these distributions are not independent, but coupled to each other.*

see that the average of the two transverse components, $x$ and $y$, is not equal to zero. This has been checked carefully with different Helmholz coils to exclude systematic errors. The asymmetric distribution of the fields becomes more apparent in Fig. 4. The larger width in the $x$-direction is due to the fact that the distribution consists of two Gaussians.

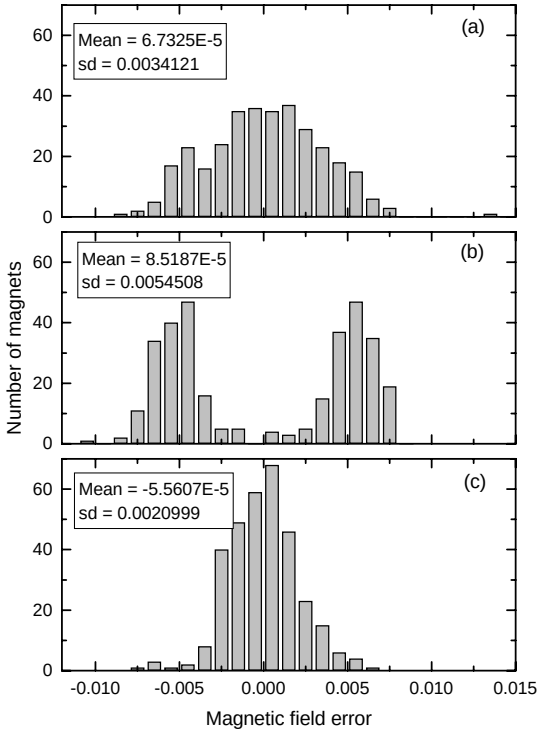



Fig. 4. *Distribution of the magnetic field for the first 326 half-periods of the initial configuration before sorting, normalized to the average undulator field. The top figure (a) gives the field for the main component, the middle figure (b) for the field in the $x$-direction, the bottom figure (c) for the $y$-direction. After redirection of the main field flux through the poles, this field will be directed in the $y$-direction.*

The electron trajectories due to these fields are shown in Fig. 5. The rms-value of the second field integral is given in the figure (where $w_{\rm rms}$ stands for the main undulator field). If one includes the first magnet pair to compensate the second field integral, hence having the electron beam on axis at the end of the undulator, all rms values become slightly less than $50\ \mu$m.

From the beam trajectories obtained with the unsorted magnets shown in Fig. 5 it is obvious that some sorting is neccessary in order to keep the beam on axis. Results of sorting after optimizing the second field integral are shown in Fig. 6. Note the angle of the

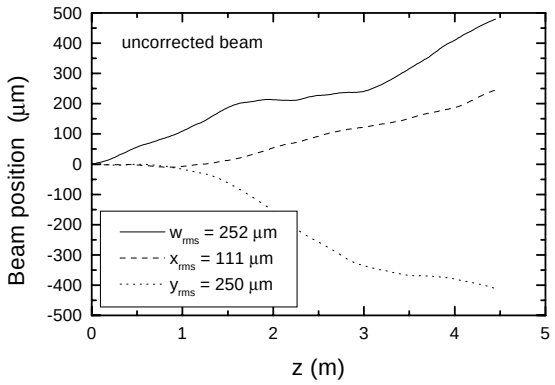



Fig. 5. *Electron beam trajectories due to the field components in Fig. 4. Calculations have been performed at an energy of 225 MeV, the minimum energy foreseen for phase I of the project.* $w_{\rm rms}$ *indicates the trajectory due to the deviation of the measured field compared to the ideal undulator field, the other two components due to the remaining error fields.*

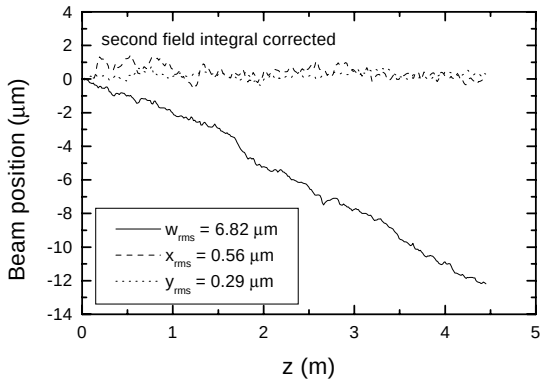



Fig. 6. *Electron beam trajectories after minimizing the second field integral. Calculations have been performed at an energy of 225 MeV, the minimum energy foreseen for phase I of the project.* $w_{\rm rms}$ *indicates the trajectory due to the deviation of the measured field compared to the ideal undulator field, the other two components due to the remaining error fields.*

undulator trajectory. This is due to the fact that the first magnet was not included in the optimization procedure of the main undulator field. Since adjustable magnets are foreseen to correct this overall initial kick the rms value is comparable to the other two. If instead of the second field integral the rms-field error would have been used as optimization criterium, the resulting second field integral would be approximately an order of magnitude larger, even if the first magnet pair is used for compensation. For the second undulator, also the first magnet has been included in the minimization procedure. Results are shown in Fig. 7.

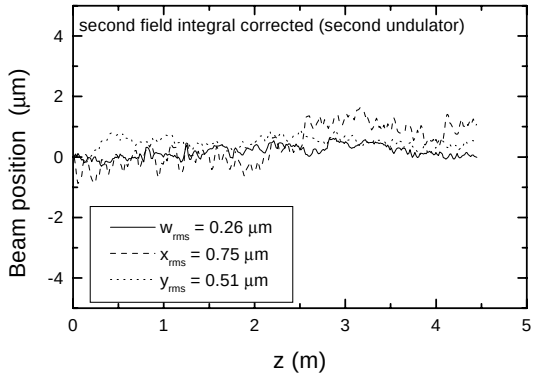



Fig. 7. *Electron beam trajectories of the second undulator after minimizing the second field integral. Calculations have been performed at an energy of 225 MeV, the minimum energy foreseen for phase I of the project.* $w_{\rm rms}$ *indicates the trajectory due to the deviation of the measured field compared to the ideal undulator field, the other two components due to the remaining error fields.*

One should note the following. Because of the distribution of the field in the $x$-direction (see Fig. 3 for the corresponding flux), one basically looses a degree of freedom. Normally, in order to get a total $x$-field equal to zero, one can choose two magnets with the same $x$-flux or opposite $x$-flux. Since the distribution is not centered around zero, the second possibility does no longer exist. Therefore, in order to minimize the $x$-component of the magnetic field, one has to choose two magnets with the same flux. In this case, a flip

of one magnet is no longer possible. Therefore, both magnets also need to have opposite $y$-components, if that is to be minimized simultaneously.

#### 5. Conclusions & Limitations

As has been shown, magnet sorting greatly improves the quality of the undulator. The simulated beam trajectories show the deviation from the ideal of only a few micrometers. This value is more than enough for the TTF-FEL, and would even be enough for the future TESLA-FEL, where the acceptable trajectory deviation goes down from 12 $\mu$m rms to typically 3 $\mu$m.

A few remarks should be made however. In calculating magnetic fields, it has been assumed that they are proportional to the magnetization of the individual magnet blocks. This assumption is no more than a first estimated guess of the actual field produced by the magnets. For example, the poles between the magnets have been assumed to be ideally placed and 100 % conducting. Hence, errors that might be caused by the poles have not been taken into account. Furthermore, the beam trajectory calculated assumes half-cosine fields per half period. The actual field is more complicated. In addition, coupling of the $B_y$ field with the undulator field has not been taken into account. The fields have been treated completely independently. However, if we assume that the poles can be adjusted accurately enough for the (undulator) field errors to become small, it is reasonable to assume that the direct fields will give a negligibly small effect. Thus the main contribution to the trajectory deviation will most certainly be the quadrupole misalignment, which has not been taken into account in any of the sorting procedures so far, even when they can be aligned with an accuracy of 10 $\mu$m.

As stated in the introduction, the field errors dominate the geometrical errors. This is probably no longer true after the magnets have been sorted. Therefore, the actual electron beam trajectories might be significantly different from those calculated assuming only the magnetic errors. After the fields have actually been measured, one should check if there is a strict relation between simulated and measured second field integrals. With this new knowledge, perhaps a refinement of the method can be obtained.

## References


[1] "*A VUV Free Electron Laser at the TESLA Test Facility: Conceptual Design Report*", DESY Print TESLA-FEL 95-03, Hamburg, DESY, 1995

[2] A.M. Kondratenko and E.L. Saldin, *Generation of Coherent Radiation by a Relativistic Electron Beam in an Ondulator*, Particle Accelerators, **10** (1980) 207.

[3] R. Bonifacio, C. Pellegrini and L.M. Narducci Optics Commun. **53** (1985) 197

[4] B. Faatz, J. Pflüger and Yu.M. Nikitina, *Study of the undulator specification for the VUV-FEL at the TESLA Test Facility*, Nucl. Instrum. Meth. **A393** (1997) 380.

[5] Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, Augusta H. Teller, Edward Teller, *Equation of State Calculations by Fast Computing Machines*, The Journal of Chemical Physics **21** (1953) 1087.

[6] S. Kirkpatric, C.D. Gelatt, Jr. M.P. Vecchi, *Optimization by Simulated Annealing*, Science **220** (1983) 671.

[7] Numerical Recipes, William H. Press, Saul A. Teukolsky, William T. Vetterling and Brian P. Flannery, Cambridge University Press, 436 ff.

[8] J. Pflüger, H. Lu and T. Teichmann, *Field fine tuning by pole height adjustment for the undulator of the TTF-FEL*, Presented at the 20th International FEL conference, August 17-21, 1998, Williamsburg, USA

[9] J. Pflüger, *Protokoll der 1. Projektbesprechung VUV-FEL Undulator I*, Internal note.

[10] B.L. Bobbs, G. Rakowsky, P. Kennedy, R.A. Cover and D.Slater In Search of a Meaningful Field-Error Specification for Wigglers Nucl. Instr. Meth. **296** (1990) 574-578

[11] Yu.M. Nikitina and J. Pflüger Influence of Magnetic field errors on the spectrum of the Petra Undulator Nucl. Instr. Meth. **A359** (1995) 89-92

[12] J. Pflüger, H. Lu, D. Köster and T. Teichmann, *Magnetic Measurements on the Undulator prototype for the VUV-FEL at the TESLA Test Facility* Nucl. Instrum. Meth. **A407** (1998) 386.