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Coherence properties of the radiation from SASE FEL

E.L. Saldin[a,*], E.A. Schneidmiller[a], M.V. Yurkov[b]

[a] *Deutsches Elektronen-Synchrotron (DESY), Notkestrasse 85, D-22607 Hamburg, Germany*
[b] *Joint Institute for Nuclear Research, Dubna, 141980 Moscow Region, Russia*

Abstract

We present a comprehensive analysis of coherence properties of radiation from a SASE FEL. The effective transverse correlation function is calculated by means of numerical simulations with the code FAST. This allows us to calculate the area and degree of transverse coherence. Evolution of these parameters is traced from the beginning of the undulator up to the deep nonlinear regime. It is shown that the degree of transverse coherence reaches maximum value at the saturation point. Then, it drops drastically because of poor transverse coherence of the radiation produced in the nonlinear regime.
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1. Introduction

The process of amplification in Self-Amplified Spontaneous Emission (SASE) FEL starts from shot noise in the electron beam. As a result, the radiation from SASE FEL has a statistical nature. During the initial stage of amplification, it has poor longitudinal and transverse coherence. Properties of the longitudinal coherence of SASE FEL radiation have been studied thoroughly in Refs. [1–7]. It has been found that the coherence time increases in the linear regime, reaches a maximum value at saturation, and then drops drastically in the deep nonlinear regime [5,7]. Recent studies of transverse coherence of SASE FEL radiation were devoted to the linear stage of amplification. It has been found that the process of formation of transverse coherence is more complicated than that given by the naive physical picture of transverse mode selection. It was found that even after finishing the transverse mode selection process, the degree of transverse coherence of the radiation from a SASE FEL visibly differs from unity [8]. This is consequence of the interdependence of the longitudinal and transverse coherence. The SASE FEL has poor longitudinal coherence which develops slowly with the undulator length, thus preventing a full transverse coherence. In this paper, we extend investigation of transverse coherence for the general case of SASE FEL operation. Our study has shown that the maximal degree of transverse coherence is achieved at saturation. Further increase of the undulator length leads only to degradation of the

*Corresponding author. Tel.: +49-40-8998-2676; fax: +49-40-8998-4475.
E-mail address: saldin@mail.desy.de (E.L. Saldin).

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properties of the radiation, despite an increasing output power. Maximal brilliance of the radiation is achieved at saturation, too.

2. General description of transverse coherence

The transverse coherence properties of the radiation are described in terms of the transverse correlation functions. The first-order transverse correlation function is defined as

$$\gamma_1(\vec{r}_\perp,\vec{r}'_\perp,z,t) = \frac{\langle \tilde{E}(\vec{r}_\perp,z,t)\tilde{E}(\vec{r}'_\perp,z,t)\rangle}{[\langle|\tilde{E}(\vec{r}_\perp,z,t)|^2\rangle\langle|\tilde{E}(\vec{r}'_\perp,z,t)|^2\rangle]^{1/2}} \quad (1)$$

where $\tilde{E}$ is the slowly varying amplitude of the amplified wave, $E_y=\tilde{E}(\vec{r}_\perp,z,t)e^{i\omega_0(z/c-t)}+\text{C.C.}$ The averaging symbol $\langle\cdots\rangle$ means the ensemble average over radiation pulses. In the following, we consider the model of a stationary process, thus assuming that γ_1 does not depend on time. The space analogue of a stationary statistical process is a model of a statistically homogeneous field. For such a field, correlation function (1) depends only on the remainder $\vec{\rho}=\vec{r}_\perp-\vec{r}'_\perp$, $\gamma_1(\vec{r}_\perp,\vec{r}'_\perp,z)=\gamma_1(\vec{\rho},z)$. Then, we introduce notion of effective correlation function

$$\gamma_1^{(\text{eff})}(\vec{\rho},z) = \frac{\int\langle\tilde{E}(\vec{R}+\vec{\rho}/2,z)\tilde{E}^*(\vec{R}-\vec{\rho}/2,z)\rangle\,d\vec{R}}{\int\langle|\tilde{E}(\vec{R},z)|^2\rangle\,d\vec{R}}. \quad (2)$$

The angular spectrum $h(\vec{k}_\perp,z)$ and the effective correlation function are connected by the Fourier transform

$$h(\vec{k}_\perp,z)=\frac{1}{(2\pi)^2}\int\gamma_1^{(\text{eff})}(\vec{\rho},z)\exp(-i\vec{k}_\perp\vec{\rho})\,d\vec{\rho}.$$

The angular spectrum $h(\vec{k}_\perp,z)$ is given by

$$h(\vec{k}_\perp,z)=\frac{\langle|A(\vec{k}_\perp,z)|^2\rangle}{\int\langle|A(\vec{k}_\perp,z)|^2\rangle\,d\vec{k}_\perp} \quad (3)$$

where $A(\vec{k}_\perp,z,t)=\int\tilde{E}(\vec{r}_\perp,z,t)\exp(-i\vec{k}_\perp\vec{r}_\perp)\,d\vec{r}_\perp$. Thus, the averaged intensity distribution in the far field $h(\vec{k}_\perp,z)$ is totally defined by the effective transverse correlation function, and vice versa. We consider a SASE FEL driven by an axisymmetric electron beam. In this case, the statistical process is isotropic: the effective correlation function depends on the modulus $\rho=|\vec{\rho}|$ and the angular spectrum depends on the modulus $k_\perp=|\vec{k}_\perp|$. Thus, we have a pair of transformations

$$h(k_\perp,z)=\frac{1}{2\pi}\int_0^\infty J_0(k_\perp\rho)\gamma_1^{(\text{eff})}(\rho,z)\rho\,d\rho \quad (4)$$

$$\gamma_1^{(\text{eff})}(\rho,z)=2\pi\int_0^\infty J_0(\rho k_\perp)h(k_\perp,z)k_\perp\,dk_\perp. \quad (5)$$

The effective radius of coherence for an isotropic inhomogeneous field is

$$r_c(z)=\left[2\int_0^\infty|\gamma_1^{(\text{eff})}(\rho,z)|^2\rho\,d\rho\right]^{1/2}, \quad (6)$$

and the area of transverse coherence is given by $S_c=\pi r_c^2$.

Number of transverse modes in the radiation beam can be defined as a ratio of total power to the power inside the area of transverse coherence [10]:

$$M=\frac{W_{\text{tot}}}{W_{\text{coh}}}=\frac{\int_0^\infty|E^2(r)|r\,dr}{\int_0^{r_c}|E^2(r)|r\,dr} \quad (7)$$

and the degree of transverse coherence is defined as $\zeta=1/M$. This is a general definition valid for any radiation field. Another possible way to define the degree of coherence, valid for linear mode of operation, is based on the statistical analysis of fluctuations of the instantaneous power W. Since in the linear regime we deal with a Gaussian random process, the power density fluctuates in accordance with the negative exponential distribution and its relative width is equal to 1. If there is full transverse coherence, then the same refers to the power. If the radiation is partially coherent, then we have a more general law for power fluctuations, namely the gamma distribution. The parameter M of this distribution can be considered as the number of transverse modes. Then, the degree of coherence in the linear regime may be defined as $\zeta=1/M=\sigma_w^2=\langle W-\langle W\rangle\rangle^2/\langle W\rangle^2$.

3. Evolution of transverse coherence in a SASE FEL

We consider an FEL amplifier with helical undulator and axisymmetric electron beam. H_w and λ_w are the amplitude of the magnetic field and the period of the undulator, respectively. The angle of the electron rotation in the undulator is equal to $\theta_s = K/\gamma$, where $\gamma = \mathscr{E}_0/m_e c^2$ is the relativistic factor of the electron with nominal energy $\mathscr{E}_0$, $K = eH_w\lambda_w/2\pi m_e c^2$ is the undulator parameter, $(-e)$ and m_e are the charge and the mass of the electron, respectively, and c is the velocity of light (we use CGS units in this paper).

We assume the transverse phase-space distribution of the particles in the beam to be Gaussian and the beam is matched to the magnetic focusing system of the undulator. The rms beam size is given by $\sigma_r = \sqrt{\varepsilon_n\beta/\gamma}$, where β is the beta function and ε_n is the rms normalized emittance. When influence of space charge and energy spread is negligible, operation of SASE FEL is described in terms of the gain parameter $\Gamma = [I\omega^2\theta_s^2/(I_A c^2\gamma_z^2\gamma)]^{1/2}$, diffraction parameter $B = 2\Gamma\sigma_r^2\omega/c$, and the efficiency parameter $\rho = c\gamma_z^2\Gamma/\omega$ [9,11]. Here $\lambda = 2\pi c/\omega$ is the radiation wavelength, I is the beam current, $I_A = mc^3/e \simeq 17$ kA is Alfven's current and $\gamma_z^2 = \gamma^2/(1+K^2)$.

To be specific, we present numerical example for diffraction parameter $B = 1$ and number of electrons in the coherence volume $N_c = I/(\rho\omega e) = 7\times 10^7$. These parameters are typical for a SASE FEL operating in the optical wavelength range. Calculations are performed with three-dimensional, time-dependent numerical simulation code FAST [12].

Fig. 1 shows the evolution of the averaged FEL efficiency versus undulator length. The averaged normalized efficiency is defined as $\langle\hat{\eta}\rangle = \langle W\rangle/(\rho P_b)$, where $\langle W\rangle$ is averaged radiation power and $P_b = \gamma mc^2 I/e$. The amplification process starts from shot noise in the electron beam, passes the stage of exponential growth, and enters nonlinear stage. The first maximum of the gain curve is usually defined as saturation point. After passing the saturation point, the efficiency of the SASE FEL continues to increase due to the growth of sidebands.

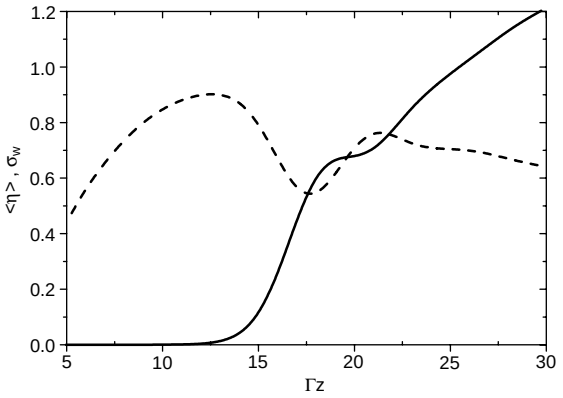


Fig. 1. Averaged efficiency of SASE FEL versus undulator length (solid line). Dashed line shows normalized rms deviation of instantaneous radiation power σ_w. Here $B = 1$, $N_c = 7\times 10^7$.

The instantaneous power of SASE FEL radiation, W, is statistical variable and fluctuates in time. Dashed line in Fig. 1 shows evolution along the undulator of normalized deviation of instantaneous radiation power. During the initial stage of amplification, fluctuations of instantaneous power are strongly suppressed, which is related to poor transverse coherence [8]. Note that, in the deep nonlinear regime we also obtain strong suppression of fluctuations. In the linear regime, probability density distribution of the radiation power is gamma distribution. When amplification process enters the nonlinear stage, the probability density distribution deviates visibly from a gamma distribution. If we trace evolution of the probability density distribution in the deep nonlinear regime, we find that it again approaches a gamma distribution. This is an indirect indication that in the deep nonlinear regime properties of the radiation again correspond to properties of completely chaotic polarized radiation. A similar asymptote was obtained also in the framework of one-dimensional model [7].

Let us go over to quantitative description of the coherence properties of radiation. The calculation procedure proceeds as follows. The numerical simulation code produces an array of field amplitudes in the near field. Using these data, we calculate the field distribution in the far field and find the averaged intensity distribution. Then, we calculate effective correlation function (5), and

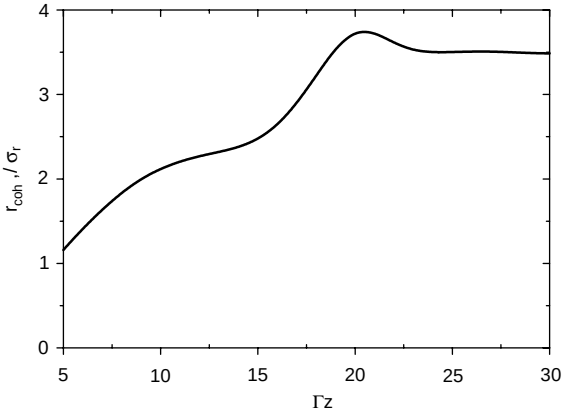


Fig. 2. Coherence radius of SASE FEL radiation versus undulator length. Here $B = 1$, $N_c = 7 \times 10^7$.

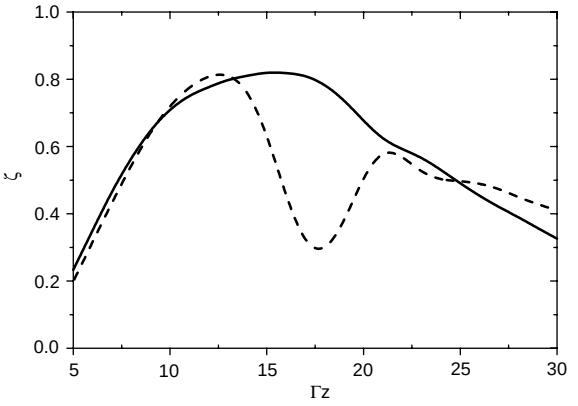


Fig. 3. Degree of transverse coherence of the radiation from SASE FEL (solid line). Dashed line shows σ_w^2 (see Fig. 1). Here $B = 1$, $N_c = 7 \times 10^7$.

radius of transverse coherence (6) (see Fig. 2). The degree of transverse coherence is given by a fraction of the radiation power passing within area of transverse coherence (7). At this stage of the algorithm the actual distribution of the radiation intensity is used. Fig. 3 shows the evolution of the degree of transverse coherence of the radiation along the undulator. Dashed curve in Fig. 3 shows evolution of σ_w^2 (see Fig. 1). As we mentioned above, the latter quantity should correspond to the degree of transverse coherence in the linear regime. Note that there is good agreement between two different physical definitions.

It is seen from Fig. 3 that maximal degree of transverse coherence is achieved at the saturation point. In the deep nonlinear regime it decreases rapidly. Comparison with the plot for averaged FEL efficiency (see Fig. 1) shows that in the deep nonlinear regime there is strict correlation between degradation of the degree of transverse coherence and increase of radiation power. This indicates that radiation produced in the nonlinear regime has poor transverse coherence. Actually, this is main message of our study. We conclude that the best quality of the radiation from SASE FEL is achieved at the saturation point. Further increase of the undulator length leads only to degradation of the properties of the radiation, despite the increase of output power. Maximal brilliance of the radiation is achieved at saturation, too.

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